HIGH SCHOOL GEOMETRY

Mathematical Practices

The Standards for Mathematical Practice are essential in the extension of mathematical thinking. Students develop these habits of mind through specific, intentional experiences of writing, reading, talking, and reasoning that connect mathematics to their daily lives and career situations. All of the Standards are important for all quality math courses:

- Make sense of problems and persevere in solving them (MP.1)
- Reason abstractly and quantitatively (MP.2)
- Construct viable arguments and critique the reasoning of others (MP.3)
- Modeling with mathematics (MP.4)
- Use appropriate tools strategically (MP.5)
- Attend to precision (MP.6)
- Look for and make use of structure (MP.7)
- Look for and express regularity in repeated reasoning (MP.8)

In Geometry and Measurement, students should recognize that much of mathematics is concerned with understanding quantities and their relationships. They should pick appropriate units for quantities being modeled, using them as a guide to understand a situation, and be attentive to the level of accuracy that is reported in a solution. The National Council of the Teachers of Mathematics (NCTM) recommends that four essential
concepts be taught during a Geometry course: Measurement; Transformations; Geometric Arguments, Reasoning, and Proof; and Solving Applied Problems and Modeling in Geometry.

Fluency 1: Measurement ● Students can use dissection and recombining to find areas and volumes of complex figures. ● Construction of measurements with different tools, including technology, helps with understanding measurement. ● Using similarity transformations, an image can be computed using proportional relationships.		
Standards	**Strands**	**Goals and Performance Objectives**
HSG.GMD.A.1	11 Circumference, Area & Volume	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
HSG.GMD.A.2	11 Circumference, Area & Volume	Give an informal argument using Cavalieri's principle for the formulas for volume of a sphere and other solid figures.

HSG.GMD.A.3	11 Circumference, Area & Volume	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
HSG.GMD.B.4	11 Circumference, Area & Volume	Identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
HSG.MG.A.1	11 Circumference, Area & Volume	Use geometric shapes, their measures, and their properties to describe objects.
HSG.MG.A.2	11 Circumference, Area & Volume	Apply concepts of density based on area and volume in modeling situations.
HSG.MG.A.3	11 Circumference, Area & Volume	Apply geometric methods to solve design problems.
HSG.GPE.B.7	3 Parallel & Perpendicular Lines 5 Congruent Triangles 7 Quadrilaterals & Other Polygons	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, using the distance formula.
HSG.C.B.5	10 Circles	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Transformations are a huge part of mathematics, both in geometry and their association with algebra. They include rigid motions such as translations, rotations, and reflections - these transformations can be used to teach congruence. Non-rigid motions like dilations can be used to teach similarity. Finally, compositions in transformations are a wonderful way to introduce function compositions that students use in algebra.

Fluency 2: Transformations
- Students can use transformations and mapping to recognize characteristics of geometric figures and describe symmetries.
- A sequence of rigid motions can be used to show congruence.
- A sequence of non-rigid motions can be used to show similarity.
- Transformations in geometry can be used a connection with algebra

Standards	Strand	Goals and Performance Objectives
HSG.CO.A.1	4 Transformations	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segments, based on the underlying notions of point, line, distance along a line, and distance around a circular arc.
HSG.CO.A.2	4 Transformations	Represent transformations in the plane using transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and given other points as outputs. Compare transformations that preserve distance and angle to those that do not.
HSG.CO.A.3	4 Transformations	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
HSG.CO.A.4	4 Transformations	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
HSG.CO.A.5	4 Transformations	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using geometry software. Specify a sequence of transformations that will carry a given figure onto another.

HSG.CO.B.7	5 Congruent Triangles	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
HSG.CO.B.8	5 Congruent Triangles	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
HSG.SRT.A.1 (a & b)	4 Transformations 8 Similarity	Verify experimentally the properties of dilations given by a center and a scale factor.
HSG.SRT.A.2	8 Similarity	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
HSG.SRT.A.3	8 Similarity	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
HSG.SRT.B.5	8 Similarity	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
HSG.SRT.C.6	8 Similarity 9 Right Triangles	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangles, leading to definitions of trigonometric ratios for acute angles.
HSG.CO.B.6	5 Congruent Triangles	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures; use the definition of congruence in terms of rigid motions to decide if they are congruent.

Mathematical proofs and Euclidean geometry go hand in hand. High school students learn to use deductive and inductive reasoning, along with logic, to build the necessary skills to complete a sufficient proof. While technology can be helpful in using transformations to make proofs more tactile for students who struggle with the format of traditional proofs. Discovering a proof can take on many methods - patty paper, dynamic software, or even manipulatives. After the students' intuition has been strategized into some process, then communicating the proof is next. This skill is extremely challenging for most students and there are various ways to proceed - two-column, paragraph, flow chart, even oral discourse.

Fluency 3: Geometric Arguments, Reasoning, and Proof

- Proofs demonstrate whether a statement is true or false mathematically, and they can be communicated in a variety of ways.
    - Technology should be used to construct and explore figures because it provides a method to explore independence and dependence of assumptions and conjectures.
- Proofs can be made with transformations, coordinates, or algebra.

Standards	Strand	Goals and Performance Objectives
HSG.GPE.B.4	2 Reasoning & Proof	Use coordinates to prove simple geometric theorems algebraically.
HSG.GPE.B.5	3 Parallel & Perp. Lines	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g. find the equation of a line parallel or perpendicular to a given line that passes through a given point)
HSG.C.A.1	10 Circles	Prove that all circles are similar
HSG.SRT.B.4	5 Congruent Triangles	Prove theorems about triangles
HSG.CO.C.9	2 Parallel & Perp. Lines	Prove theorems about lines and angles
HSG.CO.C.10	5 Congruent Triangles	Prove theorems about triangles
HSG.CO.C.11	7 Quad. & other Polygons	Prove theorems about parallelograms

Modeling in geometry is usually easier for students than most problem-solving situations. Real-life applications that involve shapes, distance, similar figures, transformations, etc. are wonderful practice for students to engage their problem solving skills.

Fluency 4: Solving Applied Problems and Modeling in Geometry ● Using congruence, similarity, symmetry, measurement, right triangle trigonometry, and other geometric ideas provide an understanding of these concepts and is a powerful tool for solving physical world problems. ● Experiencing the mathematical modeling cycle are valuable problem solving skills.		
Standards	**Strand**	**Goals and Performance Objectives**
HSG.CO.D.12	3 Parallel & Perp. Lines	Make formal geometric constructions using a variety of tools and methods
HSG.CO.D.13	10 Circles	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle

HIGH SCHOOL STATISTICS (Geometry Semester 2)

Statistics' focus on variability sets it apart from mathematics and students need to improve their statistical thinking. The wave of technology has made this area easily accessible for students. With the use of technology, such as DESMOS, students are able to learn to think critically about the data. There are 4 focus areas that the authors support in this domain - quantitative literacy, visualizing and summarizing data, statistical inference, and probability. Each focus has a prescribed list of essential concepts that should be covered extensively within its domain.

Fluency 1: Quantitative Literacy ● Statistical reasoning is crucial when students need to evaluate conclusions and assess risks. ● A quantitative literate person must be able to make and defend informed data-based decisions.		
Standards	**Strand**	**Goals and Performance Objectives**

HSS.ID.2	Statistics & Probability	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
HSS.ID.3	Statistics & Probability	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)
HSS.ID.4	Statistics & Probability	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables, and Montana American Indian data sources to estimate areas under the normal curve.

Students should be able to recognize that data traditionally is viewed in two different types - quantitative and categorical. They should also know that analyzing these types of data are not similar. Quantitative data can be interpreted in one variable with multiple forms of variation and distribution, and two variable data should be analyzed with variation correlations in order to learn about regression equations. This process is greatly enhanced by technology use and students can delve deeper into the analysis when they are able to use the technology to display the data properly.

Fluency 2: Visualizing and Summarizing Data

- Data can be viewed in two types - quantitative and categorical. Technology can be used to keep the "messiness" from interfering with the students' ability to analyze it.
- Students should be able to look at the distributions of data (continuous or discrete) and evaluate them according to shape, deviation, and outliers.
- Two categorical variables are usually represented by two-way tables or segmented bar graphs.
- Scatterplots reveal patterns, trends, clusters, and gaps and these can aid in analyzing between contextual variables ● Two quantitative variables should be analyzed according to the sum of squared deviations within linear models, patterns should be discussed in generating a least-squares regressions line and finding a correlation, and differentiating between causation and correlation. ● Data analysis techniques should be used to evaluate possible solutions to real problems.

Standards	**Strand**	**Goals and Performance Objectives**

HSS.ID.1	Statistics & Probability	Represent data with plots on the real number line (dot plots, histograms, and box plots)
HSS.ID.5	Statistics & Probability	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joining, marginal, and conditional frequencies). Recognize possible associations and trends in the data.
HSS.ID.7	Statistics & Probability	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
HSS.ID.6	Statistics & Probability	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.

It is the responsibility of mathematics teachers to train students to use the proper tools to summarize data distributions and develop the ability to statistically infer as they develop quantitative literacy.

Fluency 3: Statistical Inference

- There are 3 main types of study designs: sample survey, experiment, and observational study.
- There is a difference between randomly selected samples and in randomly assigning subjects to experimental treatment groups. ● The scope and validity of statistical inferences are dependent on the randomization in study design.
- Bias may occur in surveys and they will yield results that are not representative of the population of interest. ● The expected variability in the sampling distribution of a sample statistic is proportional to the size of the sample - larger is better. ● Repeated samples for a given sample size can lead to a plausible set of values for the general population, using margin of error to describe the sampling variability.

● Simulations can determine whether the statistic is significant or whether it is surprising or unlikely to happen under the assumption of random chance.

Standards	Strand	Goals and Performance Objectives
HSS.ID.8	Probability & Statistics	Computer (using technology) and interpret the correlation coefficient of a linear fit.
HSS.IC.1	Probability & Statistics	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
HSS.IC.2	Probability & Statistics	Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation
HSS.IC.3	Probability & Statistics	Recognize the purpose of and differences among samples surveys, experiments, and observational studies; explain how randomizations relate to each.
HSS.IC.4	Probability & Statistics	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
HSS.IC.5	Probability & Statistics	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
HSS.IC.6	Probability & Statistics	Evaluate reports based on data.

For high school students, their experiences with probability should extend beyond theoretical and empirical probabilities of outcomes. They should be able to determine independence and conditional probability, both have many applications in real-life decision making. Geometric and two way table probabilities are excellent tools for high school students to use.

Fluency 4: Probability

- Conditional probabilities can be computed by organizing data in contingency tables. Students should recognize that conditions or assumptions can affect the computation of the probability.
- The definition of independent probability events as defined by definition and also by recognizing the relationship between the events.

Standards	Strand	Goals and Performance Objectives
HSS.CP.1	Statistics & Probability	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not")
HSS.CP.2	Statistics & Probability	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
HSS.CP.3	Statistics & Probability	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
HSS.CP.4	Statistics & Probability	Construct and interpret two-way frequency tables of data including information from Montana American Indian data sources when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

HSS.CP.5	Statistics & Probability	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
HSS.CP.6	Statistics & Probability	Find the conditional probability of A given B and the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
HSS.CP.7	Statistics & Probability	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.