HIGH SCHOOL

PreCalculus (Dual Credit)

Mathematical Practices

The Standards for Mathematical Practice are essential in the extension of mathematical thinking. Students develop these habits of mind through specific, intentional experiences of writing, reading, talking, and reasoning that connect mathematics to their daily lives and career situations. All of the Standards are important for all quality math courses:

- Make sense of problems and persevere in solving them (MP.1)
- Reason abstractly and quantitatively (MP.2)
- Construct viable arguments and critique the reasoning of others (MP.3)
- Modeling with mathematics (MP.4)
- Use appropriate tools strategically (MP.5)
- Attend to precision (MP.6)
- Look for and make use of structure (MP.7)
- Look for and express regularity in repeated reasoning (MP.8)

College Algebra (Semester 1)

Sequences and Series give a cohesive look at patterns in mathematical thinking. Students will learn to recognize the patterns found in arithmetic and geometric sequences through recursive and explicit formulas. They will also learn to use the given formulas to determine the sums of series in both arithmetic and geometric, finite and infinite series. In this unit, they will also learn how to relate the binomial theorem to the Pascal triangle as well as the need for understanding as it relates to the expansion of binomials in further chapters.

<table>
<tr><td colspan="3">Content Area: Sequences & Series with Probability</td></tr>
<tr><td colspan="3">Fluency 1: Sequences & Series
Expand an given recursive and explicit formula
Use given data to write an arithmetic & geometric sequence, finding the common difference and/or ratio
Expand an explicit/recursive series, given the summation notation
Use sequences and/or series to solve real-life examples, such as annuities, growth/decay, or interest situations</td></tr>
<tr><td>Standards</td><td>Strand</td><td>Goals and Performance Objectives</td></tr>
<tr><td>HSA.SSE.B.4</td><td>8 Sequences & Series</td><td>Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.</td></tr>
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HSF.BF.A.2	8 Sequences & Series	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
HSF.LE.A.2	8 Sequences & Series	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, and a description of a relationship, or two input-output pairs (include reading these from a table)

Probability is used in many ways in real-world applications. A thorough understanding of the simple ideas of counting, permutations, and combinations is required by all competent mathematicians.

Fluency 2: Probability

- Comprehend the concepts of simple probability including events that are disjoint or overlapping. (mutually exclusive or not)
- Comprehend the concepts of simple probability including events that are independent or not.
- Understand the Fundamental Counting Theorem and use it to discover complete sets
- Distinguish between permutations and combinations, and use their formulas efficiently by technology or, if given the formula, by hand
- Use Venn diagrams to explain set-theory and probability notation - such as intersection, union, or complements

Standards	**Strand**	**Goals and Performance Objectives**
HSS.CP.A.1	8 Sequences & Series	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").
HSS.CP.A.2	8 Sequences & Series	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent
HSS.CP.A.5	8 Sequences & Series	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
HSS.CP.B.9	8 Sequences & Series	Use permutations & combinations to compute probabilities of compound events and solve problems.

Advanced algebraic concepts require abstract thought and a strong basis of functional understanding. Students are expected to be knowledgeable with linear, polynomial, exponential, logarithmic, sinusoidal, and transcendental systems, subjects covered thoroughly through Algebra II. Precalculus students quickly review these topics during semester 1 with a focus on transformations and solving algebraically for zeros of functions.

Content Area: Functions		
Fluency 1: Quadratic Equations • Using the complex number system to solve for imaginary roots in polynomials. • Completing the square is an alternative method to rewrite and solve quadratic equations. • Understanding that the parent graphs provide a strong basis to determine how transformations can affect the graph of a polynomial equation.		
Standards	**Strands**	**Goals and Performance Objectives**
HSF.BF.3	1 Functions and their Graphs	Identify the effect on the graph by replacing f(x) by f(x)+k, k f(x), f(kx), f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
HSA.CED.1	1 Functions and their Graphs	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions
HSA.CED.2	1 Functions and their Graphs	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
HS.APR.3	1 Functions and their Graphs	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
HSF.IF.4	1 Functions and their Graphs	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
HSF.IF.7	1 Functions and their Graphs	Graph equations and show intercepts, end behavior, maxima & minima.
HSF.IF.8	1 Functions and their Graphs	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
HSF.IF.9	1 Functions and their Graphs	Compare properties of two functions each represented in a different way (algebraically,

		graphically, numerically in tables, or by verbal descriptions).
HSA.REI.4	1 Functions and their Graphs	Solve quadratic equations by inspection, taking roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a+bi for real numbers a and b.
HSA.REI.7	1 Functions and their Graphs	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
HSN.CN.1	1 Functions and their Graphs	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a+bi with a and b as real numbers.
HSN.CN.2	1 Functions and their Graphs	Use the relation $i^2 = -1$, and the commutative, associative, and distributive properties to add, multiply, and subtract complex numbers.
HSN.CN.7	1 Functions and their Graphs	Solve quadratic equations with real coefficients that have complex solutions.
HSN.GPE.2	1 Functions and their Graphs	Derive the equation of a parabola given a focus and directrix.

It is critical for students to be able to simplify rational expressions once they begin Calculus. Rational expressions are an extension of the simple fractional rules and operations in a more abstract method.

Fluency 2: Rational Expressions

- Rational expressions can be simplified through factoring.
- Rational expressions can be solved through arithmetic operations, but may contain extraneous solutions.

Standards	Strand	Goals and Performance Objectives
HS.SSE.2	2 Polynomial & Rational Functions	Use the structure of an expression to identify ways to rewrite it.
HS.APR.1	2 Polynomial & Rational Functions	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication
HS.APR.6	2 Polynomial & Rational Functions	Rewrite simple rational expressions indifferent forms: where a(x), b(x), q(x), and r(x) are polynomials with degree of r(x) less than the degree of b(x) using inspection, long division, or, for more complicated examples, a computer algebra system.

HS.APR.7	2 Polynomial & Rational Functions	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Techniques to solve quadratic equations can be extended to polynomial equations. This area of advanced algebra is extremely important to melding the abstract algebra concepts to the graphical representation of polynomial functions. Students need to recognize and use the Fundamental Theorem of Algebra in this section.

Fluency 3: Polynomial Functions

- Factoring techniques can be used to solve for polynomial zeros.
- Synthetic and long division can be used to find roots of polynomial expressions.
- Graphing polynomial equations given roots and critical points.
- Identifying types of roots in a polynomial equation.

Standards	Strand	Goals and Performance Objectives
HSA.REI.1	2 Polynomial & Rational Functions	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
HSA.APR.2	2 Polynomial & Rational Functions	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a)=0 if and only if (x-a) is a factor of p(x).
HSA.APR.4	2 Polynomial & Rational Functions	Prove polynomial identities and use them to describe numerical relationships.
HSA.APR.5	2 Polynomial & Rational Functions	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$; in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example, by Pascal's triangle.
HSN.RN.1	2 Polynomial & Rational Functions	Know the Fundamental Theorem of Algebra; Show that it is true for quadratic and other polynomials.

Rewriting algebraic expressions is taught in Algebra 2 to aid in finding solutions to equations that are difficult to solve in their original form. Transferring radical equations into rational exponents is a necessary skill for Pre-Calculus.

Fluency 4: Rational and Radical Functions • Rewriting rational and radical expressions is necessary to solve equations.		
Standards	**Strand**	**Goals and Performance Objectives**
HSA.REI.A.2	2 Polynomial & Rational Functions	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise
HSN.RN.A.1	2 Polynomial & Rational Functions	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for the notation for radicals in terms of rational exponents.
HSN.RN.A.2	2 Polynomial & Rational Functions	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Logarithms & Exponential Functions (Semester 2)

Further applications of inverse functions are used in the study of logarithms and exponential expressions. Students need to be able to solve and graph both logarithmic and exponential equations.

Fluency 6: Exponential and Logarithmic Functions • Solve exponential equations using one-to-one property • Convert exponential equations into logarithmic equations • Graph exponential/logarithmic equations • Solve logarithmic equations • Use logarithms and exponents to model real-life applications, such as growth and decay examples.		
Standards	**Strand**	**Goals and Performance Objectives**
HSA.SSE.B.3c	6 Exponential & Logarithmic Functions	Use the properties of exponents to transform expressions for exponential functions.
HSF.LE.A.2	6 Exponential & Logarithmic Functions	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table)

HSF.LE.A.4	6 Exponential & Logarithmic Functions	For exponential models, express as a logarithm the solutions to an exponential equation; evaluate the logarithm using technology.
HSF.LE.B.5	6 Exponential & Logarithmic Functions	Interpret the parameters in a linear or exponential function in terms of a context.
HSF.IF.C8b	6 Exponential & Logarithmic Functions	Use the properties of exponents to interpret expressions for exponential functions.

Trigonometry (Semester 2)

Trigonometry is used to solve and model periodic functions. Students will be expected to use ratios of the sides of a right triangle as well as the angles associated with circular trigonometry.

Content Area: Trigonometry		
Fluency 1: Right Triangle Trigonometry • Use trigonometric ratios - SOH/CAH/TOA - to solve for missing sides or angles in a right triangle. • Identify and use the special right triangles' identities. • Use trigonometric ratios to solve real - life examples of right triangle work.		
Standards	**Strand**	**Goals and Performance Objectives**
HSF.TF.A.3	4 Trigonometric Functions	Use special triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi/4, and pi/6.
HSF.TF.B.7	4 Trigonometric Functions	Use inverse functions to solve trigonometric equations that arise in modeling context; evaluate the solutions using technology, and interpret them in terms of the context.
HSG.SRT.8	4 Trigonometric Functions	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Trigonometry can be applied to general triangles by using Law of Sines and Law of Cosines. Students will also learn how to use various area problems on general triangles.

Fluency 2: Analytical Geometry • Use Law of Sines and Law of Cosines to solve for oblique triangles • Find the area of oblique triangles		
Standards	**Strand**	**Goals and Performance Objectives**
HSG.SRT.D.9	6 Additional Topics in Trig	Derive the formula $A=\frac{1}{2}$ ab sin C for the area of a triangle by drawing an auxiliary line from the vertex perpendicular to the opposite side.
HSG.SRT.D.10	6 Additional Topics in Trig	Prove the Law of Cosines and Sines and use them to solve problems.
HSG.SRT.D.11	6 Additional Topics in Trig	Understand and apply the Law of Sines and Law of Cosines to find unknown measurements in right and non-right triangles (e.g. surveying problems, resultant forces)

Unit Circle Trigonometry plays a vital role in Calculus. Students need to be able to circumvent the unit circle with ease and understanding, which includes finding reference angles, coterminal angles; and easily convert degrees to radians.

Fluency 3: Unit Circle Trigonometry • Use unit circle for solve for any angle • Use function notation to graph a periodic function, including modeling of real-life applications • Convert measure of radian and degrees using unit circle		
Standards	**Strand**	**Goals and Performance Objectives**
HSF.TF.A.1	4 Trigonometric Functions	Understand radian measure of an angles as the length of the arc on the unit circle subtended by the angle
HSF.TF.A.2	4 Trigonometric Functions	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measure of angles traversed counterclockwise around the unit circle.
HSF.TF.B.5	4 Trigonometric Functions	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
HSF.TF 7	5 Analytical Trigonometry	Use inverse functions to solve trigonometric equations that arise in modeling contexts: evaluate the solutions using technology, and interpret them in terms of the context.

HSF.TF.9	5 Analytical Trigonometry	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
HSF.TF.8	4 Trigonometric Functions	Prove the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ and use it to calculate trigonometric ratios.
HSF.TF.4	4 Trigonometric Functions	Use the unit circle to explain symmetry (off and even) and periodicity of trigonometric functions.

Vectors are used in various fields of science and engineering. Students in Pre-Calculus need to be introduced to basic vector notations and operations both algebraically and graphically.

Content Area: Vectors		
Fluency 1: Vector Notation and Graphical Representation • Use vector notation appropriately • Graphically represent vectors		
Standards	**Strand**	**Goals and Performance Objectives**
HSN.VM.1	6 Additional Topics in Trig	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.
HSN.VM.2	6 Additional Topics in Trig	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point
HSN.VM.3	6 Additional Topics in Trig	Solve problems from a variety of contexts, involving velocity and other quantities that can be represented by vectors.

Fluency 2: Vector Arithmetic • Add and subtract vectors. • Multiply a vector by a scalar.		
HSN.VM.4a	6 Additional Topics in Trig	Add vectors end to end, component wise, and by the parallelogram rule. Understand that the magnitude of the sum of two vectors is typically not the sum of the magnitudes.
HSN.VM.4b	6 Additional Topics in Trig	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
HSN.VM.4c	6 Additional Topics in Trig	Understand vector subtraction **v–w** as **v+(–w)** where **–w** is the additive inverse of **w**, with the same magnitude and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component wise.
HSN.VM.5a	6 Additional Topics in Trig	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component wise.
HSN.VM.5b	6 Additional Topics in Trig	Compute the magnitude of a scalar multiple c**v** using $\|\|c\mathbf{v}\|\| = \|c\|\mathbf{v}$. Compute the direction of c**v** knowing that when $\|c\|\mathbf{v}$ is not equal to 0, the direction of c**v** is either along **v** or against **v**.